Journal of Environmental Sciences Vol.5, No. 4, pp. 435–443, 1993 ISSN 1001-0742

Prediction for emission of trace gases by GM(1,1) model*

Yang Wenxiang

Research Center for Eco-Environmental Sciences,
Chinese Academy of Sciences, Beijing 100085, China

Abstract— This paper recommend the grey control system model (GM1, 1) to approach the problem of prediction for emission of trace gases. When the number of data in the system is not enough for mathermatics statistics analysis, the application of GM(1, 1) model can get available results. This model can be used for prediction of emission trace gases and trend of global concentration of CO_2. The atmospheric concentration of CO_2 will be 365.9 ppmv by the year 2000. The prediction for emission of CO_2 from buming coal will be 1123.3 TgC in China. The emissions of CO in China are estimated.

Keywords: trace gases; GM(1, 1) model; burning caol; greenhous effect.

1 Introduction

The impact of human activity on environment has attracted more attention. The increasing of atmospheric temperature by greenhouse effect, the depletion of ozone layer and climate change, all of these may be relate to the trace gases which are emitted into atmosphere by anthropogenic activity. Obviously, the emissions of trace gases are restricted by many complicated factors. These systems can be treated by probabilistic method, multi-regression analysis and multi-factor analysis and so on. All of those analytical methods can get available results if we have enough number of data. But if the number of data can not meet the need for statistics analysis, the results would be uncertainty. The determinations of concentration of atmospheric trace gases have been carried out accurately only since the sixtieth. Rightnow, some of the analytical methods are still needed to be improved. Therefore the data are limited in number. Now we try to introduce the grey control system model to predict the emission of atmospheric trace gases. The grey control system was developed by Prof. Dang (Dang, 1988). Now we are going to recommend the basic of the grey control system model as well as the prediction for emission of trace gases by GM (1, 1) model in the following.

* This project is supported by National Natural Science Foundation of China

2 Basic of GM(1, 1) model (Dang, 1988)

In order to establish GM(1,1) model, a series of number X^0 is needed.

Let $X^0 = [X^0\ (1),\ X^0\ (2),\ \cdots\cdots\cdots\cdots X^0\ (n)]$.

The series of number can be selected from the experimental data and/or the statistical data. These data are fluctuating in a definite range. Some of the factors which cause the variation of the data are known, but some of the factors are unknown. In order to find out the regular patterns of this series of data, it is treated as following.

This series of number is treated by 1-accumulated generating operation.

$$X^1\ (k) = \sum_{i=1}^{k} X^0(i),$$

so

$X^1(1) = X^0(1)$,

$X^1(2) = X^0(1) + X^0(2)$.

$\cdots\cdots\cdots\cdots\cdots\cdots$

$X^1(n) = X^0(1) + X^0(2) + \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots + X^0(n)$.

Then a series of new number X^1 is formed.

$X^1 = [X^1(1),\ X^1\ (2), \cdots\cdots\cdots\cdots, X^1(n)]$.

It seems that the fluctuation of this new series number is more smooth than the original one. So the dynamic differential equation can be established.

$$\frac{dX^1}{dt} + aX^1 = u$$

This is a GM(1, 1) model differential equation with one order and one variable. A series of parameter is $\hat{a}$.

$$\hat{a} = \begin{bmatrix} a \\ u \end{bmatrix}.$$

Using the least square method, we can get $\hat{a}$.

$$\hat{a}=(B^T\cdot B)^{-1}\cdot B^T\cdot \bar{Y}_n$$

Here

$$B=\begin{bmatrix} -\frac{1}{2}(X^1(1)+X^1(2)), & 1 \\ -\frac{1}{2}(X^1(2)+X^1(3)), & 1 \\ \cdots\cdots\cdots & \\ -\frac{1}{2}(X^1(n-1)+X^1(n)), & 1 \end{bmatrix}$$

$$\bar{Y}_n=[X^0(2),X^0(3),\cdots\cdots\cdots,X^0(n)]^T.$$

So, the solution of this differential equation is

$$\hat{X}^1(k+1)=(\hat{X}^0(1)-\frac{u}{a})\cdot\exp(-ak)+\frac{u}{a}.$$

Then according to inverse accumulated general operation, we can get $\hat{X}^0$.

$$\hat{X}^0(k)=\hat{X}^1(k)-\hat{X}^1(k-1).$$

$$\hat{X}^0=[\hat{X}^0(1),\ \hat{X}^0(2),\cdots\cdots\cdots,\hat{X}^0(n)]$$

This series of number is the modeling calculated values. The modeling calculated values are compared with experimental values. The error can be determined. The precision of this model is tested by post error test. If the precision of this model is good, the model is available for prediction. If the precision of the modle is lower than the good level, the error will be treated by GM(1,1) model once again and the calculated values need to reform.

Now let us definite:

$$C=\frac{S_1}{S_2}$$

$$P=P(|q(k)-\bar{q}|)<0.6745S1.$$

Where, C is the ratio of post error; P is the frequence of least error; $q(k)$ is the residue error at k; S_1^2 is the square error of original data; $S_1^2 = \frac{1}{n}\sum_{i=1}^{n}(X(k)-\bar{X})^2$; S_2^2 is the square error of residue error; $S_2^2 = \frac{1}{n}\sum_{i=1}^{n}(q(k)-\bar{q})^2$.

The precision of model can be qualified by P and C (Table 1).

Table 1 The degrees of precision of the model

	P	C
Good	>0.95	<0.35
Qualified	>0.80	<0.50
Just	>0.70	<0.65
Unqualified	<0.70	>0.65

The advantages of GM(1,1) model are: when the number of data is not enough for mathematics statistics, probabilistic and regression analysis, the application of GM(1,1) model can get good results; GM(1,1) model can be used for multifunction system; the operation of GM(1,1) model is easy. We have designed a programme for the operation of GM(1,1).

3 The prediction for the emission of CO_2 from burning fossil fuel

The consumption of fossil fuel in China during 1980 – 1988 was reported in "The Year Book of Statistic in China-1989" (National Statistic Agency of China, 1989). We estimated the emission of CO_2 from combustion of fossil fuel by using the method of Rotty and Marland (Rotty, 1986). The results are shown in Table 2.

Table 2 The amounts of emission of CO_2 from combustion of fossil fuel

Years	Emission of CO_2 TgC	Years	Emission CO_2, TgC
1980	396.0	1985	482.1
1981	371.1	1986	509.1
1982	388.1	1987	546.2
1983	412.0	1988	577.4
1984	455.1		

Note: Selected the data from 1983 to 1987, a series of number X^0 is formed.
$X^0 = [X^0(1), X^0(2), \cdots\cdots, X^0(5)] = (412.0,\ 455.1,\ 482.1\ 509.1,\ 546.2)$

This series of number is treated by 1-accumulated generating operation.

$$X^1(k)=\sum_{i=1}^{k} X^0(i), \qquad i=1,\cdots\cdots, 5.$$

Then we get a new series of number X^1.

A figure can be draw by using these data. It is obvious that the new figure is more smooth than that of the original one. So a dynamic differential equation can be established.

The solution of this differential equations is

$$\hat{a}=[\begin{matrix} a \\ u \end{matrix}].$$

where, $a=-0.0604886$; $u=415.2125$.

$$\hat{X}^1(k+1)=7276.3017 \exp(0.0604886k)-6864.3017.$$

Then a series of calculated data $\hat{X}^0$ is given by inverse accumulated generating operation.

$$\hat{X}^0(k)=\hat{X}^1(k)-\hat{X}^1(k-1).$$

$$\hat{X}^0=(412.0,\ 453.7,\ 482.0,\ 512.0,\ 544.0,\ 577.9).$$

The precision of this model was tested by posterror test method. $P=1$; $C=0.03843$. So the precision of this model is good enough for prediction. The calculated data and experimental data are shown in Table 3.

Table 3 The prediction for emission of CO_2 from burning fossil fuel in China

Years	X^0 determinated values, TgC	$\hat{X}^0$ Calculated values, TgC	q%
1980	396.0		
1981	371.7		
1982	388.10		
1983	412.0	412.0	0.0
1984	455.1	453.7	+0.30
1985	482.1	482.0	+0.02
1986	509.1	512.1	−0.59
1987	546.2	544.0	+0.40
1988	577.5	577.9	−0.07
1989		613.9	
1990		652.2	
2000		1194.2	

Let X^0 being the experimental data collected from 1980 to 1985, and the data are treated as the above procedure. Then we can get another series of calculated data.

It is interesting that we can get different values of a and u from different original point. In fact, according to the data collected in the reseasonable period, we can get more reliable results.

4 Some examples for application of GM(1,1) model

4.1 The prediction for the global atmospheric concentration of CO_2

The atmospheric concentrations of CO_2 are reported in the UNEP Environmental Data Reports 1989/1990 (UNEP, 1990). We selected the data in the period from 1965 to 1987. These data are treated by GM(1,1) model using a *PC* program. The X^1 equation is given in the following:

$$\hat{X}^1(k+1)=81411.0897 \exp(0.00392593k)-81090.6797,$$

$$P=1;\ C=0.060106.$$

Then the values of atmospheric concentration of CO_2 are calculated. The calculated values are compared with the experimental values in Table 4. It is obvious that the residue error is in the range from -0.3% to $+0.27\%$. The precision of this GM(1,1) model, qualified by values of P and C, is good enough for prediction. It indicated that the concentration of CO_2 will be 365.97 ppmv by the year 2000. This result is similar to the result reported by Lashof (Lashof, 1989).

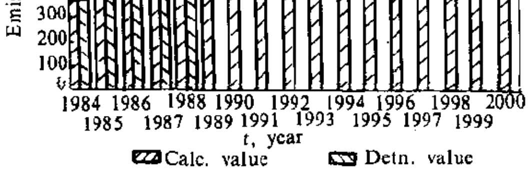

Fig. 1 Emission of CO_2 from burning coal

4.2 The prediction for the emission of CO_2 from burning coal in China

The consumption of coal was reported in the Yearbook of National Statistic Agency of China (Statistic Agency of China, 1989). According to the methods of Marland, we can estimate the emission of CO_2 from burning coal in China (Yang, 1993). Then we selected the data from 1983 to 1987, and input it in GM(1,1) model. So, we have got a equation as following:

$$\hat{X}^1(k+1)=5173.00\exp(0.0690316k)-4832.8,$$

$$P=1;\ C=0.05909.$$

The emission of CO_2 from burning coal can be calculated. The calculated values and the determined values are shown in Fig. 1. Here $q\%$ is in the range from -1.38% to 1.13%. The precision of this model is very good.

Table 4 The prediction for atmospheric concentrations of CO_2

Years	Determined values, ppm[1]	Calculated values, ppm	Q%
1965	320.41	320.41	–
1966	321.09	320.24	0.27
1967	321.90	321.51	0.12
1968	322.72	322.76	−0.01
1969	324.21	324.03	0.05
1970	325.51	325.32	0.06
1971	326.48	326.58	−0.03
1972	327.60	327.87	−0.08
1973	329.83	329.16	0.14
1974	330.41	330.46	−0.01
1975	331.01	331.76	−0.23
1976	332.06	333.06	−0.30
1977	333.63	334.37	−0.23
1978	335.19	335.68	−0.15
1979	336.54	337.01	−0.14
1980	338.40	338.34	0.20
1981	339.46	339.66	−0.06
1982	340.76	341.00	0.07
1983	342.76	342.34	0.12
1984	344.34	343.69	0.19
1985	345.65	345.05	0.17
1986	346.84	346.39	0.13
1987	348.62	347.76	0.25
1988		349.14	
1989		350.05	
1990		351.88	
1991		353.27	
1992		354.65	
1993		356.04	
1994		357.46	
1995		358.86	
2000		365.97	

1. From UNEP Environmental Data Report 1989/1990

$\hat{X}^1(k+1) = 81411.0897\exp(0.0039359k) - 81090.6797$,

$P=1$; $C=0.060109$.

The prediction for the emission of CO_2 from burning coal in China will be 1123.3 TgC by the year 2000.

4.3 Emission of CO from burning coal in China

It was reported that the emission factor of CO from burning coal is 45.0 kg/ton (Cullis, 1989). Therefore, we can set up a GM(1,1) model. The equation of $\hat{X}^1$ is established as follows:

$$\hat{X}^1(k+1)=261.99285\exp(0.0710056k)-242.41285,$$

$P=1$; $C=0.08188$.

The calculated values and determinated values are presented in Fig.2. The precision of this GM(1,1) model is good.

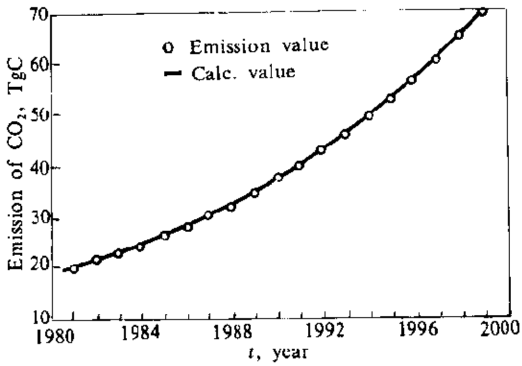

Fig.2 Emission of CO from burning coal

5 Conclusions

5.1 It was shown that GM(1,1) model can be used for prediction of emission of atmospheric trace gases and trend of global concentration of trace gases. The advantage of GM(1,1) is remarkable. When the number of data is not enough for mathematics statistics, probabilistic analysis, the application of GM(1,1) model can get good results. Especially, the operation of GM(1,1) model is easy.

5.2 The prediction for global atmospheric concentration of CO_2 will be 365.97 ppmv by the year 2000. The emission of CO_2 from burning coal will be 1123.3 TgC in China. The emission of CO in China are estimated.

References

National Statistic Agency of China. The year book of statistic in China-1989. Beijing: National Statistics Press, 1989:351

Cullis CF, Hirshler MM. Atmospheric Environment, 1989; 23(6):1195

Dang Jilong. The basic method of grey control system. Wuhan: Huazhong Technology University Press, 1988

Lashof D. Climate Change, 1989; 14:213

Rotty RM, Marland G. A global analysis (Ed by Trabalka JR, Reichle RE). Springer-Verlag, 1986:474

UNEP. UNEP environmental data report 1989/1990
Yang Wenxiang. Estimating the emission of some trace gases in China (this issue)

(Received December 23, 1992)