

Study on the wind field and pollutant dispersion in street canyons using a stable numerical method

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Abstract: A stable finite element method for the time dependent Navier-Stokes equations was used for studying the wind flow and pollutant dispersion within street canyons. A three-step fractional method was used to solve the velocity field and the pressure field separately from the governing equations. The Streamline Upwind Petrov-Galerkin (SUPG) method was used to get stable numerical results. Numerical oscillation was minimized and satisfactory results can be obtained for flows at high Reynolds numbers. Simulating the flow over a square cylinder within a wide range of Reynolds numbers validates the wind field model. The Strouhal numbers obtained from the numerical simulation had a good agreement with those obtained from experiment. The wind field model developed in the present study is applied to simulate more complex flow phenomena in street canyons with two different building configurations. The results indicated that the flow at rooftop of buildings might not be assumed parallel to the ground as some numerical modelers did. A counter-clockwise rotating vortex may be found in street canyons with an inflow from the left to right. In addition, increasing building height can increase velocity fluctuations in the street canyon under certain circumstances, which facilitate pollutant dispersion. At high Reynolds numbers, the flow regimes in street canyons do not change with inflow velocity.

Keywords: finite element method; Streamline Upwind Petrov-Galerkin method; three-step fractional method

Introduction

Studying the flow pattern in street canyons is of great interests to fluid modelers, environmental scientists as well as urban planners because it helps us a better understanding of the complex pollutant dispersion behaviors inside a street canyon. Flows and related phenomena are mathematically governed by a set of nonlinear partial differential equations. Difficulties have been experienced in obtaining numerical solutions of the Navier-Stokes equations at high Reynolds number, which mainly due to two reasons. Firstly, the governing equations become advection-dominated at high Reynolds number. The conventional numerical methods will lead to spurious oscillatory solutions for flow problems at high Reynolds numbers. Secondly, the velocity and the pressure unknowns are coupled in the governing equations, which will typically result in a large number of unknowns. Even for two-dimensional problems with moderate geometrical complexity, the coupled approach requires large amount of memory and excessive computation time (CPU time). Therefore, the coupled approach with direct solvers appears not feasible. It is a challenging work to deploy stable and cost-effective numerical methods to solve the governing equations.

In this study, a three-step fractional method is used to solve the velocity field and the pressure field separately from the governing equations following the numerical schemes proposed by Hughes *et al.* (Hughes, 1986) and Xia and Leung (Xia, 2001a; 2001b). The Streamline Upwind Petrov-Galerkin (SUPG) technique (Hughes, 1986) is introduced to prevent numerical oscillations for flows at high Reynolds numbers. The LU pre-conditioner together with the Successive Over Relaxation (SOR) method (Press, 1992) is used to solve the large sparse system (Xia, 2000).

The quality of the numerical model is verified by simulating the flow across a square cylinder under different Reynolds numbers and the computed result is compared with the experimental data of Okajima (Okajima, 1982). The wind field model is then applied to simulate more complex flow phenomena in street canyons. Two typical building configurations, i.e. multiple identical buildings and multiple non-identical buildings, are used in the simulations.

1 Numerical scheme

1.1 Wind field model

The continuity equation and the momentum equations, that mathematically govern the incompressible viscous flow, are expressed in non-dimensional velocity pressure form as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

and

$$\begin{aligned} \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= -\frac{\partial p}{\partial x} + \frac{1}{Re} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \\ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} &= -\frac{\partial p}{\partial y} + \frac{1}{Re} \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right), \end{aligned} \quad (2)$$

where u , v represent the velocity components; p for pressure; Re is the Reynolds number; and t is the time.

Intensive computing resources are required for the numerical solution of the coupled system. One convenient way to reduce the requirement of computer system is to solve the velocity field and pressure field separately from the momentum equations and continuity equation. This idea was first pioneered by Chorin (Chorin, 1968) being as the fractional step method and was further extended to the finite element method by Donea *et al.* (Donea, 1982), Hughes *et al.* (Hughes, 1986), Xia and Leung (Xia, 2001a; 2001b). The velocity field and pressure field at each time step are solved numerically through the following three steps:

- Compute the intermediate velocities u and v from the momentum Equations (2) with the pressure terms eliminated;
- Solve the pressure field, which is initially ignored, from the pressure Poisson equation;
- Update the velocities to satisfy both the momentum and the continuity equation.

The eight-node isoparametric element with bi-quadratic interpolation for velocity and bi-linear interpolation for pressure over an element is used to discretize the space domain. The temporal domain is discretized using the Euler's scheme. The convective terms are discretized using the second order Adams-Bashforth scheme. The Streamline Upwind Petrov-Galerkin (SUPG) technique (Hughes, 1986)

is introduced to prevent numerical oscillations for flows at high Reynolds numbers. Detailed implementation of numerical scheme can be found in Xia (Xia, 2000). The numerical scheme for the wind field model is proved cost-effective for solving velocity and pressure fields from the Navier-Stokes equations.

1.2 Particle trajectory model

A Lagrangian particle dispersion model is used to describe the pollutant transport phenomena within the street canyons(Xia, 2001b). Pollutant trajectories are predicted by discharging a large amount of particles into the computation domain. It is noted that the same grid system was adopted in computing the wind field and pollutant dispersion. The grid close to the buildings and the ground is refined to account for strong gradients.

2 Results and discussion

2.1 Numerical comparison

The Strouhal number of a simulated flow across a square cylinder under different Reynolds numbers is examined and compared with those experimental data of Okajima(Okajima, 1982) to verify the quality of the numerical model. Fig.1 summarizes the variation of Strouhal number with Reynolds number, both from the numerical simulations and experimental data. It can be seen that the variation of Strouhal number in the present numerical simulations has overall agreement with that of the experiment. Lower-frequency and larger amplitude velocity fluctuations become dominant when the Reynolds number is beyond 100, largely due to the sudden change of flow regimes. The present numerical model can correctly capture the change of laminar flow to transient flow regime starting at about $Re = 100$. For the numerical simulations, the Strouhal number increases slightly from 0.15 to 0.17 while the Reynolds number increases from 80 to 100. The Strouhal number drops to 0.12 when the Reynolds number further increases to 300 and varies between 0.11 and 0.12 thereafter, which is in close agreement with the data of Okajima(Okajima, 1982).

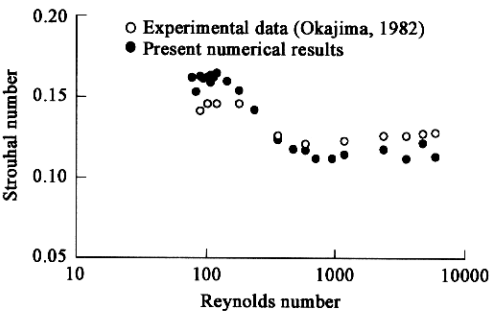


Fig.1 Variation of Strouhal number with Reynolds number

2.2 Numerical simulations

The wind field model is then applied to simulate more complex flow phenomena in street canyons. Cases of four identical buildings and multiple non-identical buildings are selected for the simulations. Three interesting findings are obtained from the present study, which worth noticing for dispersion modelers. As can be seen in Fig.2, the flow at rooftop of each of the building is quite different from one to another and may not be assumed parallel to the ground as some numerical modelers did. Furthermore, three different flow regimes are formed in the three similar street canyons from upstream to downstream. A fully developed counter-

clockwise rotating vortex is found in C01 with an inflow from left to right. This is due to the separation of the incident flow at the front corner of the upstream building, generating a clockwise rotating vortex in the wake area above the street canyon, and driving a counter-clockwise rotating vortex in the street canyon. Another counter-clockwise rotating vortex is formed in C02. However, it is suppressed and constrained in the street canyon due to a strong clock-wise rotating vortex above the street canyon. In contrast to the above, a clockwise-rotating vortex is found in C03, which is consistent with the result of wind tunnel studies. This is because the flow separated from the windward corner of B01 starts to re-attach at the rooftop of C03, resulting in a change in flow direction which is consistent with the free stream flow. It has been found previously that the flow velocity at the rooftop would become parallel to the ground and strong shear flow would be generated while the number of upstream identical buildings increases. As a consequence, a clockwise-rotating vortex is generated in street canyons when the testing buildings are all identical, as it does in C03. Actually, this is what has been observed in previous wind tunnel experiments(Gerdes, 1999). Increasing the height of some buildings can increase velocity fluctuations in the street canyon under certain circumstances; which facilitate pollutant dispersion(Fig. 3). This phenomenon is not reported in previous wind tunnel experiments largely because they focused on the effect of the height and width of street canyons on flow patterns. The buildings in most of the experiments were set identical and may not reflect the complex building configurations in real world. It reveals a fact that the target street canyon cannot be isolated from the surrounding building settings when a dispersion model in urban street canyons is concerned. At high Reynolds numbers, the flow regimes remain unchanged in street canyons when inflow velocities U_o increase from 0.1 m/s to 6.0 m/s(Fig.4). A strong counter-clockwise rotating vortex is formed in all testing inflow velocities with the normalized maximum wind velocities U/U_o in street canyons maintaining at a constant value (0.53 ± 0.05) (Table 1), indicating that wind velocities in street canyons increases proportionally with the inflow velocity.

Table 1 Normalized maximum flow velocities(U_{max}/U_o) in street canyons ($G/H = 1.5$) at different inflow velocities(U_o)

U_o , m/s	0.1	0.2	0.3	0.5	0.75	1.5	3.0	6.0
U_{max}/U_o	0.54	0.57	0.54	0.52	0.54	0.51	0.48	0.55

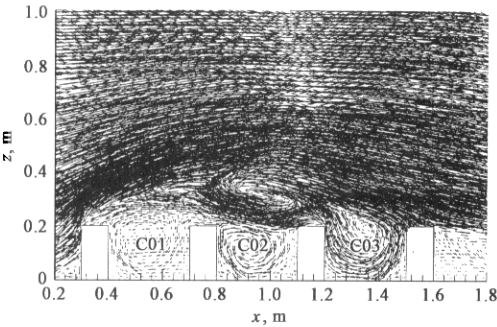


Fig.2 Velocity field around four identical buildings

3 Conclusions

A stable finite element method for the time dependent Navier-Stokes equations is presented for the case of flow over

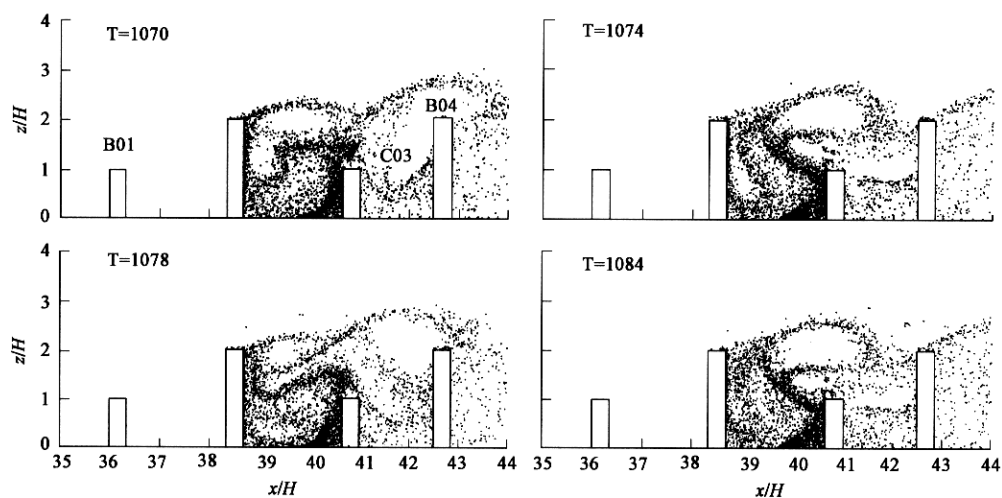


Fig.3 Pollutant dispersion in a street canyon surrounded by high buildings

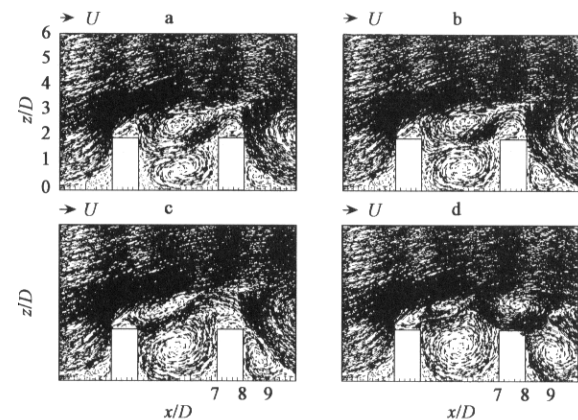


Fig.4 Wind field around two identical buildings at different inflow velocities
a. $U_0 = 0.1$ m/s; b. $U_0 = 0.75$ m/s; c. $U_0 = 1.5$ m/s; d. $U_0 = 6.0$ m/s

street canyons. A wind field model and a particle trajectory model have been developed and applied to simulate the complex flow phenomena in street canyons. Flows over two typical building configurations are simulated and three interesting findings are obtained: (1) The flow at rooftop of buildings may not be assumed parallel to the ground as some numerical modelers did. A counter-clockwise rotating vortex may be found in street canyons with an inflow from the left to right; (2) Increasing building height can increase velocity fluctuations in the street canyon under certain circumstances, which facilitate pollutant dispersion; (3) At high Reynolds

numbers, the flow regimes in street canyons do not change with inflow velocity.

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