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Entropy method for determination of weight of evaluating indicators in fuzzy synthetic evaluation for water quality assessment

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Abstract: Considering the difficulty of fuzzy synthetic evaluation method in calculation of the multiple factors and ignorance of the relationship among evaluating objects, a new weight evaluation process using entropy method was introduced. This improved method for determination of weight of the evaluating indicators was applied in water quality assessment of the Three Gorges reservoir area. The results showed that this method was favorable for fuzzy synthetic evaluation when there were more than one evaluating objects. One calculation was enough for calculating every monitoring point. Compared with the original evaluation method, the method predigested the fuzzy synthetic evaluation process greatly and the evaluation results are more reasonable.

Keywords: water quality evaluation; fuzzy; weight of evaluating indicator; entropy method

Introduction

The fuzzy synthetic evaluation for water quality assessment of nature water resource has been studied and put into practice extensively in recent years (Qin, 2003; Liang and Jiang, 2002; Zhu and Lv *et al.*, 1994). By using the fuzzy mathematics theory, the evaluation results of the fuzzy synthetic evaluation would be more scientific and reasonable than the normal methods. It takes into the consideration of the fuzzy characteristic of the graduate change from low to high level of the water pollution degree. It is necessary to determine every main factor affecting the water quality, together with setting evaluating indicators, evaluating set, and membership function. Then the weight of every indicator and degree of membership are calculated to get the synthetic degree of membership so as to determine the water quality level.

The design of weight is one of the important parts in the fuzzy synthetic evaluation, as it would have a deep effect on the evaluation results. In the typical method of fuzzy synthetic evaluation, the determination of the weight of every indicator is by calculating the superscale which is the ratio of the value of every indicators at each monitoring point over corresponding water quality standard (Fan, 1998). There are some limits of the application of fuzzy synthetic evaluation in water quality assessment. The weight of every evaluating indicator should be calculated under each evaluating object. When there are multiple evaluating objects, the workload would be too heavy. Also, the value of weight from the calculation only contains the information of the individual indicator, which has nothing to do with the relationship among evaluating objects.

To solve the above problems, the entropy method could be used in water quality assessment. It firstly appeared in thermodynamics, and was introduced into the information theory later by Shannon (1948). Nowadays, it has been widely used in engineering, economy, finance, etc. (Tian and Du, 2004; Chen and Zhang, 2003; Guo, 2001; Zhao and Song, 2001; Fang *et al.*, 2004; Li *et al.*, 2004; Zhou, 2003; Xu *et al.*, 2004; Tang *et al.*, 2000; Lin *et al.*, 2003). Information entropy is the measurement of the disorder degree of a system (Meng, 1989). It can measure the amount of useful information with the data provided. When the difference of the value among the evaluating objects on the same indicator is high, while the entropy is small, it illustrates that this indicator provides more useful information, and the weight of this indicator should be set correspondingly high. On the other hand, if the difference is smaller and the entropy is higher, the relative weight would be smaller (Qiu, 2002). Hence, the entropy theory is an objective way for weight determination. The determination of weight by calculating entropy is to choose the best indicators which could reflect the different pollution level among monitoring points. This paper used the entropy method to determine the weight of evaluating indicators in fuzzy synthetic evaluation, and applied it in water quality evaluation in city sections of the Three Gorges reservoir area.

1 Assessment method (Qiu, 2002)

1.1 Normalization of the original evaluating matrix

Suppose there are evaluating indicators counted m, evaluating objects counted n, then forms an original indicators value matrix $X=(x_{ij})_{m\times n}$,

$$X=\begin{vmatrix} x_{11}, x_{12}, \cdots, x_{1n} \\ x_{21}, x_{22}, \cdots, x_{2n} \\ \\ x_{m1}, x_{m2}, \cdots, x_{mn} \end{vmatrix} \quad (1)$$

Normalization this matrix to get Equation (2):

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$$R=(r_{ij})_{m\times n} \tag{2}$$

where, r_{ij} is the data of the jth evaluating object on the indicator, and $r_{ij}\in[0,1]$. Among these indicators, to which the bigger the better, there are

$$r_{ij}=\frac{x_{ij}-\min\limits_{j}\{x_{ij}\}}{\max\limits_{j}\{x_{ij}\}-\min\limits_{j}\{x_{ij}\}} \tag{3}$$

while, the smaller the better, there are

$$r_{ij}=\frac{\max\limits_{j}\{x_{ij}\}-x_{ij}}{\max\limits_{j}\{x_{ij}\}-\min\limits_{j}\{x_{ij}\}} \tag{4}$$

1.2 Definition of the entropy

In the m indicators, n evaluating objects evaluation problem, the entropy of ith indicator is defined as

$$H_i=-k\sum_{j=1}^{n}f_{ij}\ln f_{ij},\ i=1,2,\cdots,m \tag{5}$$

In which $f_{ij}=r_{ij}\Big/\sum\limits_{j=1}^{n}r_{ij}$, $k=1/\ln n$, and suppose when $f_{ij}=0$, $f_{ij}\ln f_{ij}=0$.

1.3 Definition of the weight of entropy

The weight of entropy of ith indicator could be defined as:

$$w_i=\frac{1-H_i}{m-\sum\limits_{i=1}^{m}H_i} \tag{6}$$

in which $0\leqslant w_i\leqslant 1$, $\sum\limits_{i=1}^{m}w_i=1$.

2 Assessment

2.1 Evaluating objects and data source

In this paper, the data from 13 monitoring sections in the main city sections of the Three Gorges reservoir area were used for the study of fuzzy synthetic evaluation for water quality evaluation. Fig.1 shows all the water quality monitoring data from literature (Liu and Qu, 2002).

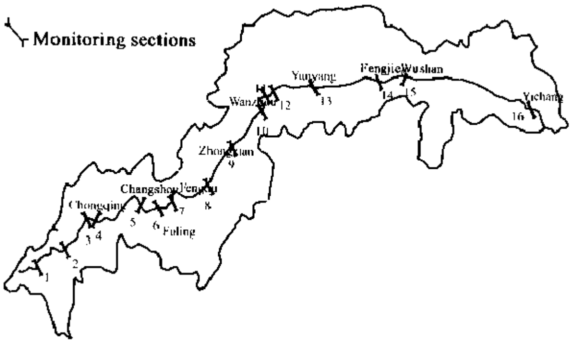

Fig.1 Location of the monitoring sections in the main city sections of the Three Gorges reservoir area
1. Baisha; 2. Huangtan; 3. Wanglongmen; 4. Cuntan; 5. Huangcaoxia; 6. Yazuishi; 7. Meimuze; 8. Mishiquan; 9. Jiutiaogou; 10. Tuokou; 11. Hongshaze; 12. Shaiwangba; 13. Yanmatou

2.2 Evaluating indicators and set

In the original monitoring data, there are dozen of monitoring indicators. After selection, some monitoring indices, which can not be evaluated directly in the water quality standard like pH, NO_2^-, NO_3^-, etc., were deleted. Further more, all the values of monitoring sections below the limitation of water quality I standard were deleted, such as NH_3, volatile hydroxybenzene, As, Hg, Pb, Cd, etc. In the end, DO, COD_{Cr}, BOD_5, TP were selected as the evaluating indicators, $U=\{DO, COD_{Cr}, BOD_5, TP\}$. According to Environmental quality standards for surface water (GB3838-2002, State Environmental Protection Administration of China, 2002), the water quality is distinguished as 5 degrees, hence, the evaluating set is $V=\{I, II, III, IV, V\}$. The values of the above 4 evaluating factors in 13 monitoring sections are shown in Table 1.

Table 1 Measured values of the 4 indicators in the monitoring sections (Liu and Qu, 2002)

Monitoring section	DO, mg/L	COD_{Cr}, mg/L	BOD_5, mg/L	TP, mg/L
Baisha	7.5	8.4	0.9	0.075
Huangtan	8.0	8.7	2.1	0.326
Wanglongmen	8.0	12.0	1.2	0.204
Cuntan	8.0	12.5	1.3	0.177
Huangcaoxia	7.5	9.9	2.3	0.120
Yazuishi	7.7	6.6	1.9	0.175
Meinvze	8.0	6.4	1.7	0.187
Mishiquan	7.2	11.1	1.6	0.147
Jiutiaogou	7.7	15.3	2.3	0.060
Tuokou	7.2	12.6	2.0	0.110
Hongshaze	7.5	14.1	1.9	0.120
Shaiwangba	7.6	14.8	2.0	0.120
Yanmatou	7.8	8.6	1.0	0.120

2.3 Formation of single-indicator evaluating matrix

According to the determined limitation of the state standard, membership function of every indicator belongs to every water quality degree would be written out (Zhu and Lv, 1994; Pan *et al.*, 2002; Hu *et al.*, 2000; Qiu *et al.*, 2003). Thus, single-indicator membership degree can be confirmed to get the single-indicator evaluating matrix R. Take Baisha monitoring section as an example, its membership matrix is

$$R=\begin{vmatrix} 0.47 & 0.53 & 0 & 0 & 0 & DO & 6.7 \\ 0.5 & 0.5 & 0 & 0 & 0 & COD_{Cr} & 9.2 \\ 0.5 & 0.5 & 0 & 0 & 0 & BOD_5 & 0.9 \\ 0.05 & 0.95 & 0 & 0 & 0 & TP & 0.096 \end{vmatrix} \tag{7}$$

The other membership matrix of the monitoring sections would not be shown here.

2.4 Calculation of the weight

First of all, normalize the original matrix from monitoring data (Table 2). The entropy and its weight could be calculated according to the data of the entropy weight (Table 3).

Table 2 Initialization of the original data from Table 1

Monitoring section	DO, mg/L	COD_{Cr}, mg/L	BOD_5, mg/L	TP, mg/L
Baisha	0.105	0.708	1.000	0.655
Huangtan	0.474	0.308	0.000	0.195
Wanglongmen	0.368	0.083	0.579	0.000
Cuntan	0.316	0.000	0.526	0.164
Huangcaoxia	0.316	0.125	0.105	0.425
Yazuishi	0.158	0.917	0.368	0.473
Meinvze	0.474	1.000	0.474	0.451
Mishiquan	0.368	0.600	0.263	0.447
Jiutiaogou	0.842	0.067	0.158	1.000
Tuokou	0.632	0.683	0.368	0.451
Hongshaze	1.000	0.392	0.368	0.465
Shaiwangba	0.947	0.333	0.263	0.429
Yanmatou	0.000	0.642	0.895	0.429

Table 3 Entropies and weights of the indicators

Evaluating indicator	Entropy	Weight
DO	0.9056	0.2538
COD_{Cr}	0.8835	0.3132
BOD_5	0.9052	0.2549
TP	0.9338	0.1781

2.5 Evaluation results

The results of fuzzy synthetic evaluation is shown in Table 4. By the entropy method for determination of weight of evaluating indicators, the result implies that the most water quality degrees in monitoring sections in the main city sections of the Three Gorges reservoir area are level I. This illustrates the water body is still clean and the extent of combined pollution is not severe. Among all the evaluating indicators, COD_{Cr} has a bigger weight that there is a more severe pollution of oxygen consumption organic substance in the main city sections of the Three Gorges reservoir area. According to the definition of weight of entropy, many monitoring sections have great difference on this indicator, for example, the water in Huangqian, Wanglongmen, Cuntan, which in city Chongqing area, has a much severe industry pollution, while relatively better in water area of Changshoujiang.

Table 4 Results of the synthetic evaluation

Monitoring section	Grade of membership					Level
	Level I	Level II	Level III	Level IV	Level V	
Baisha	0.411	0.589	0.000	0.000	0.000	II
Huangtan	0.521	0.301	0.178	0.000	0.000	I
Wanglongmen	0.434	0.282	0.206	0.078	0.000	I
Cuntan	0.386	0.267	0.335	0.012	0.000	I
Huangcaoxia	0.433	0.407	0.161	0.000	0.000	I
Yazuishi	0.419	0.515	0.066	0.000	0.000	II
Meinvze	0.521	0.404	0.075	0.000	0.000	I
Mishiquan	0.487	0.436	0.077	0.000	0.000	I
Jiutiaogou	0.656	0.225	0.119	0.000	0.000	I
Tuokou	0.538	0.387	0.075	0.000	0.000	I
Hongshaze	0.538	0.393	0.069	0.000	0.000	I
Shaiwangba	0.538	0.378	0.084	0.000	0.000	I
Yanmatou	0.369	0.453	0.084	0.000	0.000	II

2.6 Comparison with the typical fuzzy synthetic evaluation

The entropy method and the typical fuzzy synthetic evaluation was compared, so as to validate the effectiveness of the new method (Table 5). The outcome indicates that the entropy method has somewhat the same evaluation result with the traditional method. The weight of evaluating indicators calculated by the traditional method is shown in Table 6. In the traditional way of calculating weight, values of the above 13 monitoring sections data have been used 13 times. Besides, to the same value of monitoring section with the same indicator, the weight determined may have quite different value. For example, the monitoring values of Wanglongmen and Mishiquan are the same of 7.2, but the weight determined is separately 0.198 and 0.247, with which a big difference of 20%. The error is quite serious.

Table 5 Comparison of different approaches

Monitoring section	BS	HT	WLM	CT	HCX	YZS	MNZ	MSQ	JTG	TK	HSZ	SWZ	YMT
Entropy method	II	I	I	I	I	II	I	I	I	I	I	I	II
Traditional method	II	I	I	III	II	II	I	II	I	I	I	I	II

Notes: BS. Baisha; HT. Huangtan; WLM. Wanglongmen; CT. Cuntan; HCX. Huangcaoxia; YZS. Yazuishi; MNZ. Meinvze; MSQ. Mishiquan. JTG. Jiutiaogou; TK. Tuokou; HSZ. Hongshaze; SWZ. Shaiwangze; YMT. Yanmatou

Table 6 Weights by traditional approach

Monitoring section	BS	HT	WLM	CT	HCX	YZS	MNZ	MSQ	JTG	TK	HSZ	SWZ	YMT
DO	0.329	0.202	0.198	0.206	0.235	0.267	0.261	0.247	0.238	0.224	0.202	0.204	0.277
COD_{Cr}	0.180	0.186	0.211	0.230	0.239	0.108	0.098	0.166	0.289	0.145	0.196	0.206	0.159
BOD_5	0.081	0.171	0.099	0.108	0.177	0.156	0.150	0.168	0.197	0.148	0.146	0.160	0.081
TP	0.221	0.312	0.362	0.316	0.257	0.260	0.286	0.266	0.036	0.255	0.246	0.260	0.276

Notes: The same as Table 5

3 Conclusions

Comparing the traditional fuzzy synthetic evaluation method and the entropy method for determination of weight of evaluating indicators, a big decrease of workload in the evaluation has achieved. With the typical method, when making evaluation for water quality of more than 13 monitoring sections, one need to calculate once at every monitoring sections to get the weight of 4 evaluating indicators, totally 13 times of repeated work. However, using the entropy method in determination of weight, only one calculation is made to get a set of weight suited for all the monitoring sections. When N monitoring points are in existence, the workload could be decrease to $1/N$ times of the original one in the process of determining weight.

In the traditional fuzzy synthetic evaluation, the weight of evaluating indicators is determined by the monitoring data compared to water quality standard. As a result, when an abnormal value appears at some evaluating indicator, the condition of overestimate of weight of this indicator would lead to bad evaluation result. The entropy method for determination of weight considers adequately the information of values all the monitoring sections provided to balance the relationship among numerous evaluating objects. This weakens the bad effect from some abnormal values and makes the result of evaluation more accurate and reasonable. The evaluation result of the example indicates that the entropy method for determination of weight is a very effective method for evaluating indicators. Thus, it is important in application of fuzzy synthetic evaluation for water quality.

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